Determination of Expected Profit for Newsboy for Uniform Demand

Assume that demand is from a normal distribution with mean and standard deviation given by \( \mu, \sigma \). Let the sales price be \( p \), the salvage price be \( s \), and the item cost be \( c \).

\( \Pi(Q) \) is the expected profit for the newsboy from ordering \( Q \) units.

\( \phi(x|\mu,\sigma) \) denotes the probability density function for a normal distribution with parameters \( \mu, \sigma \).

\[
\Pi(Q) = \int_{-\infty}^{Q} (px + s(Q-x))\phi(x|\mu,\sigma)dx + \int_{Q}^{\infty} (pQ)\phi(x|\mu,\sigma)dx - cQ
\]

**Explanation:**
- The first integral is over the demand realizations that are less than the order quantity \( Q \); if demand equals \( x \) and if \( x < Q \), then the newsboy will sell \( x \) units at price \( p \) and salvage \( (Q-x) \) units at \( s \).
- The second integral is over the demand realizations that are more than the order quantity \( Q \); in these cases, the newsboy can only sell \( Q \) units at price \( p \).
- The last term is what the newsboy pays for ordering \( Q \) units.

To evaluate this expression, we re-write as follows:

\[
\Pi(Q) = \int_{-\infty}^{Q} (px + s(Q-x))\phi(x|\mu,\sigma)dx + \int_{Q}^{\infty} (pQ)\phi(x|\mu,\sigma)dx - cQ
\]

\[
= \int_{-\infty}^{Q} (px + s(Q-x))\phi(x|\mu,\sigma)dx + \int_{Q}^{\infty} (px + s(Q-x))\phi(x|\mu,\sigma)dx - \int_{Q}^{\infty} (px + s(Q-x))\phi(x|\mu,\sigma)dx
\]

\[
+ \int_{Q}^{\infty} (pQ)\phi(x|\mu,\sigma)dx - cQ
\]

\[
= \int_{-\infty}^{\infty} (px + s(Q-x))\phi(x|\mu,\sigma)dx - \int_{Q}^{\infty} (p-s)(x-Q)\phi(x|\mu,\sigma)dx - cQ
\]

We can then simplify this as follows:

\[
\Pi(Q) = p\mu + s(Q - \mu) - (p-s)\int_{Q}^{\infty} (x-Q)\phi(x|\mu,\sigma)dx - cQ
\]
Now the crux of the evaluation is to evaluate the third integral; for normal distribution the following can be shown, directly from algebraic transformations:

\[
\int_{x=Q}^{x=\infty} (x - Q) \phi(x \mid \mu, \sigma) \, dx = \sigma \int_{x=z}^{x=\infty} (x - z) \phi(x \mid 0,1) \, dx
\]

where \( z = \frac{Q - \mu}{\sigma} \)

Note that \( \phi(x \mid 0,1) \) is the probability density function for the standard normal distributed random variable with mean of 0 and standard deviation of 1.

The above expression is known as the partial loss function and can be calculated as follows:

\[
\text{PartialLossFunction}(z) = \int_{x=Q}^{x=\infty} (x - z) \phi(x \mid 0,1) \, dx = \phi(z \mid 0,1) - z \times (1 - \Phi(z))
\]

\[
= \text{NORMDIST}(z,0,1,\text{FALSE}) - z \times (1 - \text{NORMDIST}(z,0,1,\text{TRUE}))
\]

Where \( \Phi(z) \) is the cumulative distribution function for the standard normal. The spreadsheet commands are shown for calculation.

Thus the newsboy profit calculation is

\[
\Pi(Q) = p\mu + s(Q - \mu) - (p - s) \text{PartialLossFunction}\left(z = \frac{Q - \mu}{\sigma}\right) - cQ
\]