Tangent approximation

1. Find the equation of the tangent plane to the graph of \( z = xy^2 \) at the point (1,1,1).

   **Answer:** \( \frac{\partial z}{\partial x} = y^2 \) and \( \frac{\partial z}{\partial y} = 2xy \) \( \Rightarrow \) \( \frac{\partial z}{\partial x}(1, 1) = 1 \) and \( \frac{\partial z}{\partial y}(1, 1) = 1. \)

   The tangent plane at (1,1,1) is
   \[
   (z - 1) = \left. \frac{\partial z}{\partial x} \right|_0 (x - 1) + \left. \frac{\partial z}{\partial y} \right|_0 (y - 1) = (x - 1) + 2(y - 1).
   \]

2. Give the linearization of \( f(x, y) = e^x + x + y \) at (0,0).

   **Answer:** The tangent approximation formula at the point \((x_0, y_0, z_0)\) is
   \[
   f(x, y) - f(x_0, y_0) \approx f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).
   \]
   (We usually abbreviate this as \( \Delta z \approx f_x|_0 \Delta x + f_y|_0 \Delta y.\))

   Linearization is just the following form of the tangent approximation formula
   \[
   f(x, y) \approx f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).
   \]

   In our case,
   \[
   f_x(x, y) = e^x + 1 \text{ and } f_y(x, y) = 1 \Rightarrow f(0, 0) = 1, f_x(0, 0) = 2, f_y(0, 0) = 1
   \]

   Thus, for \((x, y) \approx (0, 0)\) we have
   \[
   f(x, y) \approx 1 + 2x + y.
   \]