Partial derivatives

1. Let $f(x, y) = e^{(x^2+y^2)} + x^2 + y^2 + xy + 2y + 3$.

   a) Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

   b) Show the second partials can be computed in any order. That is,

   $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

   c) Find $\frac{\partial f}{\partial x}(1, 3)$.

   Answer: a) $\frac{\partial f}{\partial x} = 2xe^{(x^2+y^2)} + 2x + y$, $\frac{\partial f}{\partial y} = 2ye^{(x^2+y^2)} + 2y + x + 2$.

   b) To compute $\frac{\partial^2 f}{\partial x \partial y}$ we compute the partial with respect to $x$ of $\frac{\partial f}{\partial y}$.

   $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial x} \left( 2ye^{(x^2+y^2)} + 2y + x + 2 \right) = 4xye^{(x^2+y^2)} + 1$.

   Likewise

   $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial y} \left( 2xe^{(x^2+y^2)} + 2x + y \right) = 4xye^{(x^2+y^2)} + 1$.

   We have shown the order of differentiation didn’t matter.

   c) Evaluating $\frac{\partial f}{\partial x}(1, 3) = 2e^{10} + 5$. 