Product rule for vector derivatives

1. If $r_1(t)$ and $r_2(t)$ are two parametric curves show the product rule for derivatives holds for the dot product.

**Answer:** This will follow from the usual product rule in single variable calculus. Let’s assume the curves are in the plane. The proof would be exactly the same for curves in space. We want to prove that

$$\frac{d(r_1 \cdot r_2)}{dt} = r_1' \cdot r_2 + r_1 \cdot r_2'.$$

Let $r_1 = \langle x_1, y_1 \rangle$ and $r_2 = \langle x_2, y_2 \rangle$. We have,

$$r_1 \cdot r_2 = x_1x_2 + y_1y_2.$$

Taking derivatives using the product rule from single variable calculus, we get

$$\frac{d(r_1 \cdot r_2)}{dt} = \frac{d(x_1x_2 + y_1y_2)}{dt} = x_1'x_2 + x_1x_2' + y_1'y_2 + y_1y_2' = (x_1'x_2 + y_1'y_2) + (x_1x_2' + y_1y_2') = \langle x_1', y_1' \rangle \cdot \langle x_2, y_2 \rangle + \langle x_1, y_1 \rangle \cdot \langle x_2', y_2' \rangle = r_1' \cdot r_2 + r_1 \cdot r_2'. \quad \Box$$