**Intersection of a line and a plane**

1. Find the intersection of the line through the points \((1, 3, 0)\) and \((1, 2, 4)\) with the plane through the points \((0, 0, 0)\), \((1, 1, 0)\) and \((0, 1, 1)\).

**Answer:** This brings together a number of things we’ve learned. We must find the equations of the line and the plane and then find the intersection.

The basic data specifying a line are a point and a direction. We have

\[ P_0 = (1, 3, 0) \quad \text{and} \quad v = \langle 1, 2, 4 \rangle - \langle 1, 3, 0 \rangle = \langle 0, -1, 4 \rangle. \]

Therefore, the equations for the line are

\[ x = 1, \quad y = 3 - t, \quad z = 4t. \]

The basic data specifying a plane are a point and a normal vector. We have

\[ Q_0 = (0, 0, 0) \quad \text{and} \quad N = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \langle 1, -1, 1 \rangle. \]

Therefore the equation of the plane is

\[ x - y + z = 0. \]

Substituting the line equations into the plane equation gives

\[ 1 - (3 - t) + 4t = 0 \iff t = \frac{2}{5}. \]

We use this to find the point of intersection

\[ (x, y, z) = (1, \frac{13}{5}, \frac{8}{5}). \]