Geometry of systems of equations

1. Write a 3-by-3 system of equations
   a) with no solutions and where all the planes are parallel;
   b) where two planes are parallel and the other intersects them;
   c) where the planes are all different and all intersect in a line.

   **Answer:** a) Planes are parallel if there normals are parallel. Here are two examples of such a system. We show a sketch of the second one.

   \[
   \begin{align*}
   x + 2y + 3z &= 5 & \quad z &= 0 \\
   x + 2y + 3z &= 7 & \quad z &= 2 \\
   x + 2y + 3z &= 9 & \quad z &= 4
   \end{align*}
   \]

   b) If planes are not parallel then they intersect, so it is easy to find many examples of this. Here are two, with a sketch of the second one.

   \[
   \begin{align*}
   x + 2y + 3z &= 5 & \quad z &= 1 \\
   x + 2y + 3z &= 7 & \quad z &= 3 \\
   x + y + z &= 0 & \quad x &= 0
   \end{align*}
   \]

   c) This is a little trickier. We’ll use a lot of zeros to help. The following system intersects in the z-axis

   \[
   \begin{align*}
   x &= 0 \\
   y &= 0 \\
   x + y &= 0
   \end{align*}
   \]