18.02 Practice Exam 4B

**Problem 1.** (10 points)
Let \( C \) be the portion of the cylinder \( x^2 + y^2 \leq 1 \) lying in the first octant \( (x \geq 0, y \geq 0, z \geq 0) \) and below the plane \( z = 1 \). Set up a triple integral in cylindrical coordinates which gives the moment of inertia of \( C \) about the \( z \)-axis; assume the density to be \( \delta = 1 \).

(Give integrand and limits of integration, but do not evaluate.)

**Problem 2.** (20 points: 5, 5, 10)

a) A solid sphere \( S \) of radius \( a \) is placed above the xy-plane so it is tangent at the origin and its diameter lies along the \( z \)-axis. Give its equation in spherical coordinates.

b) Give the equation of the horizontal plane \( z = a \) in spherical coordinates.

c) Set up a triple integral in spherical coordinates which gives the volume of the portion of the sphere \( S \) lying above the plane \( z = a \). (Give integrand and limits of integration, but do not evaluate.)

**Problem 3.** (20 points: 5, 15)

Let \( \vec{F} = (2xy + z^3) \hat{i} + (x^2 + 2yz) \hat{j} + (y^2 + 3xz^2 - 1) \hat{k} \).

a) Show that \( \vec{F} \) is conservative.

b) Using a systematic method, find a potential function \( f(x, y, z) \) such that \( \vec{F} = \nabla f \). Show your work, even if you can do it mentally.

**Problem 4.** (25 points: 15, 10)

Let \( S \) be the surface formed by the part of the paraboloid \( z = 1 - x^2 - y^2 \) lying above the xy-plane, and let \( \vec{F} = x \hat{i} + y \hat{j} + 2(1 - z) \hat{k} \).

Calculate the flux of \( \vec{F} \) across \( S \), taking the upward direction as the one for which the flux is positive. Do this in two ways:

a) by direct calculation of \( \iint_S \vec{F} \cdot \hat{n} \, dS \);

b) by computing the flux of \( \vec{F} \) across a simpler surface and using the divergence theorem.

**Problem 5.** (25 points: 10, 8, 7)

Let \( \vec{F} = -2xz \hat{i} + y^2 \hat{k} \).

a) Calculate curl \( \vec{F} \).

b) Show that \( \iint_R \text{curl} \vec{F} \cdot \hat{n} \, dS = 0 \) for any finite portion \( R \) of the unit sphere \( x^2 + y^2 + z^2 = 1 \). (take the normal vector \( \hat{n} \) pointing outward).

c) Show that \( \oint_C \vec{F} \cdot d\vec{r} = 0 \) for any simple closed curve \( C \) on the unit sphere \( x^2 + y^2 + z^2 = 1 \).